Kinetic theory of gases

S.I. unit of R

From perfect gas equation

$PV = nRT$

$$R = \frac{PV}{nT} = \frac{\text{unit of } P \times \text{unit of } V}{\text{unit of } n \times \text{unit of } T}$$

$$= \frac{N m^{-2} \times m^3}{mole \times K}$$

$$= \frac{N m}{mole\, K}$$

$$\boxed{R = J\, mole^{-1}\, K^{-1}}$$

value of R

For 1 mole of a perfect gas

$PV = RT$

or $R = \frac{PV}{T}$

At normal temperature and normal pressure (NTP)

$T = 273 K$

$P = h\rho g$

$= .76m \times 13.6 \times 10^3 kg/m^3 \times 9.8 m/sec^2$

$= 1.013 \times 10^5 N m^{-2}$

The volume of one mole of a gas is 22.4 litre

$= 22.4 \times 10^{-3} m^3$

$$R = \frac{1.013 \times 10^5 \times 22.4 \times 10^{-3}}{273}$$

$$\boxed{R = 8.31\, J\, mole^{-1}\, k^{-1}}$$

Perfect gas equation :→ A gas which fulfils the following conditions is known as perfect gas or ideal gas

(i) the molecules of the gas are point masses

(ii) the intermolecular forces among the molecules are zero.

(iii) the gas obeys gas law i.e Boyle's law, Charle's law and pressure law.

Numericals.

① Calculate the r.m.s velocity of hydrogen at N.T.P. Given, density of hydrogen = 0.09 $Kg\,m^{-3}$

Solution :→ $\rho = 0.09\ Kg\,m^{-3}$

$P = 76\,cm$ of mercury

$= 0.76 \times 13.6 \times 10^3 \times 9.8$

$= 1.01 \times 10^5\ N\,m^{-2}$

$$C = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{0.09}}$$

$$= \sqrt{3.37 \times 10^6} = 1.836 \times 10^3\ m/sec.$$

$$= 1836\ m/sec.$$

② At what temperature, will the root mean-square velocity of hydrogen be double of its value at S.T.P, pressure remaining constant.

Solution :→ $T_1 = 273K.$ $C_2 = 2C_1$

$$\frac{C_1}{C_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{C_1^2}{C_2^2} = \frac{T_1}{T_2}$$

$$T_2 = T_1\left(\frac{C_2^2}{C_1^2}\right)$$

$$= 273 \times (2)^2 = 273 \times 4 = 1092K.$$

$$= 1092 - 273$$

$$= 819^{\circ}C.$$

③ Calculate the rms velocity of oxygen molecule at 27°C, atomic weight of oxygen is 16.

Solution :→ $T = 27^{\circ}C = 273 + 27 = 300K.$

Atomic weight of oxygen = 16

∴ Molecular weight of oxygen = 2×16 = 32

∴ $M = 32$

$R = 8.31\ J \cdot mol^{-1}\,K^{-1}$

$$C = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 300}{32}}$$

$$= 483.5\ m/sec.$$

(4) Calculate the molecular kinetic energy of a unit mass of helium at NTP. What will be its kinetic energy at $100^\circ C$?

Solution :→ Molecular weight of helium, $M = 4$

$R = 8.31\ J\ mol^{-1}\ K^{-1}$, $T = 273K$

K.E per gram molecule $= \frac{3}{2}RT$

∴ Molecular K.E of unit mass

$$= \frac{3RT}{2M} = \frac{3 \times 8.31 \times 273}{2 \times 4}$$

$= 850.7$ Joule.

Molecular K.E at $100^\circ C$

$$= \frac{3}{2} \cdot \frac{RT}{M} = \frac{3 \times 8.31 \times 373}{2 \times 4}$$

$= 1162$ Joule.

(5) Calculate the K.E of a gram molecule of argon at $127^\circ C$. Boltzmann's constant, $K = 1.381 \times 10^{-23}\ J\ molecule^{-1}\ C^{-1}$ and Avogadro's number $= 6.022 \times 10^{23}$ molecules mol^{-1}

Solution :→ $T = 127^\circ C = 273 + 127 = 400K$.

$K = 1.381 \times 10^{-23}\ J\ molecule^{-1}\ K^{-1}$

$N = 6.022 \times 10^{23}$ molecules mol^{-1}

K.E per gram molecule of argon $= \frac{3}{2}RT$

But $R = KN$

∴ $K.E = \frac{3}{2}NKT$

$$= \frac{3}{2} \times 1.381 \times 10^{-23} \times 6.022 \times 10^{23} \times 400$$

$= 4989.83\ J.$